Uncertainty of b-value Estimation in Connection with Magnitude Distribution Properties of Small Data Sets

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Uncertainty of *b*-value Estimation in Connection with Magnitude Distribution Properties of Small Data Sets

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Summary

In this study we evaluate the efficiency of the commonly used maximum likelihood estimator of the Gutenberg-Righter (G-R) *b*-value, as it was introduced by Aki (1965). We use synthetic data sets which exhibit diverse but well defined properties. The deviation of the estimated *b* parameter from its real value is quantified by Monte Carlo simulations as a function of catalogue features and data properties, e.g. the sample size, the distribution of magnitude uncertainties, the round-off interval of the reported magnitude values and the magnitude range (ΔM). Within the objective of this study, algorithms have been compiled for the determination of such observational-theoretical deviations and to facilitate the construction of nomograms corresponding to diverse cases of input parameters. In this way, a more accurate estimation of the uncertainty level for the *b*-value and Magnitude of Completeness (M_C) determination can be achieved, contributing to a more robust seismic hazard assessment, especially at low activity areas and induced seismicity sites. Such estimations may also be particularly relevant to evaluate the *b*-value uncertainties in moving time windows which comprise small number of events.

Our results indicate that a b-value analysis, especially for small data sets, should be carried out together with a ΔM analysis. Nomograms should be constructed and adjusted to each particular case study in order to achieve a more accurate estimation of the b-value and its corresponding uncertainty. Improper b-value and M_C evaluation may in turn lead to significant miscalculation of the actual seismicity rates and seismic hazard parameters (Leptokaropoulos et al., 2018).

Synthetic Data

Assumptions:

- Each synthetic catalogue comprises a complete part which follows the G-R law (with diverse b-values) and an incomplete part corresponding to a specified detection level
- Magnitudes are rounded to their first decimal
- The noise assigned to the original synthetic catalogue is magnitude independent and normally distributed (μ=0, σ:0.0-0.4):

$$M' = M + \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(M-\mu)^2}{2\sigma^2}\right]$$

Complete part of the magnitude distribution:

$$f(M) = \frac{\beta\left[\exp(-\beta(M - M_{\min} + \Delta M/2))\right]}{1-\exp\left[-\beta(M_{\max} - M_{\min} + \Delta M/2)\right]} \quad for \quad M_{\min} \le M \le M_{\max},\ 0 \quad otherwise$$

$$F(M) = \begin{cases} 0 & for \quad M < M_{\min} \\ \dfrac{1-\exp\left[(-\beta(M - M_{\min} + \Delta M/2))\right]}{1-\exp\left[-\beta(M_{\max} - M_{\min} + \Delta M/2)\right]} & for \quad M_{\min} \le M \le M_{\max} \\ 1 & for \quad M > M_{\max} \end{cases} \qquad \beta = b\ln 10$$

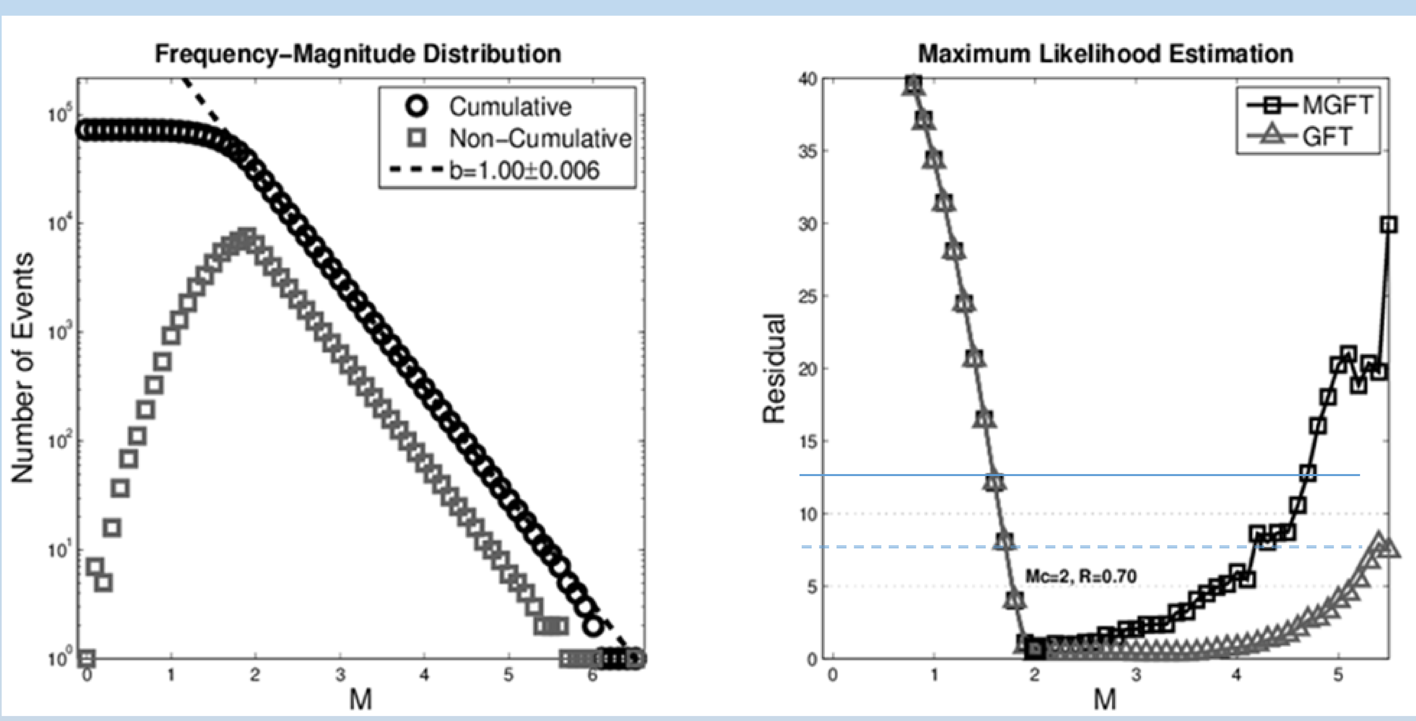
Incomplete part of the magnitude distribution:

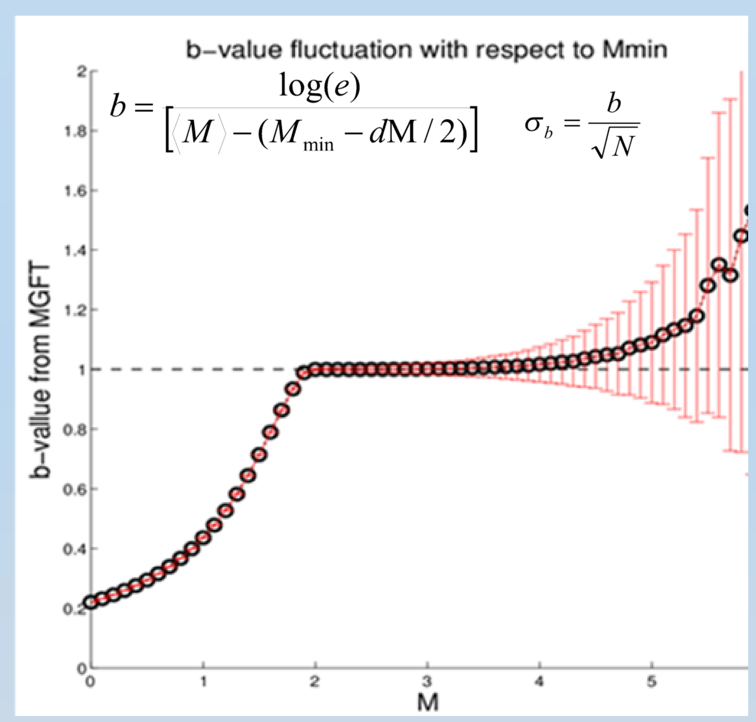
$$q(M \mid \mu, \sigma) = \begin{cases} \dfrac{1}{\sigma\sqrt{2\pi}} \displaystyle\int_{-\infty}^{M_C} \exp\left[-\frac{(M-\mu)^2}{2\sigma^2}\right] dM, & M < M_C \\ 1, & else \end{cases}$$

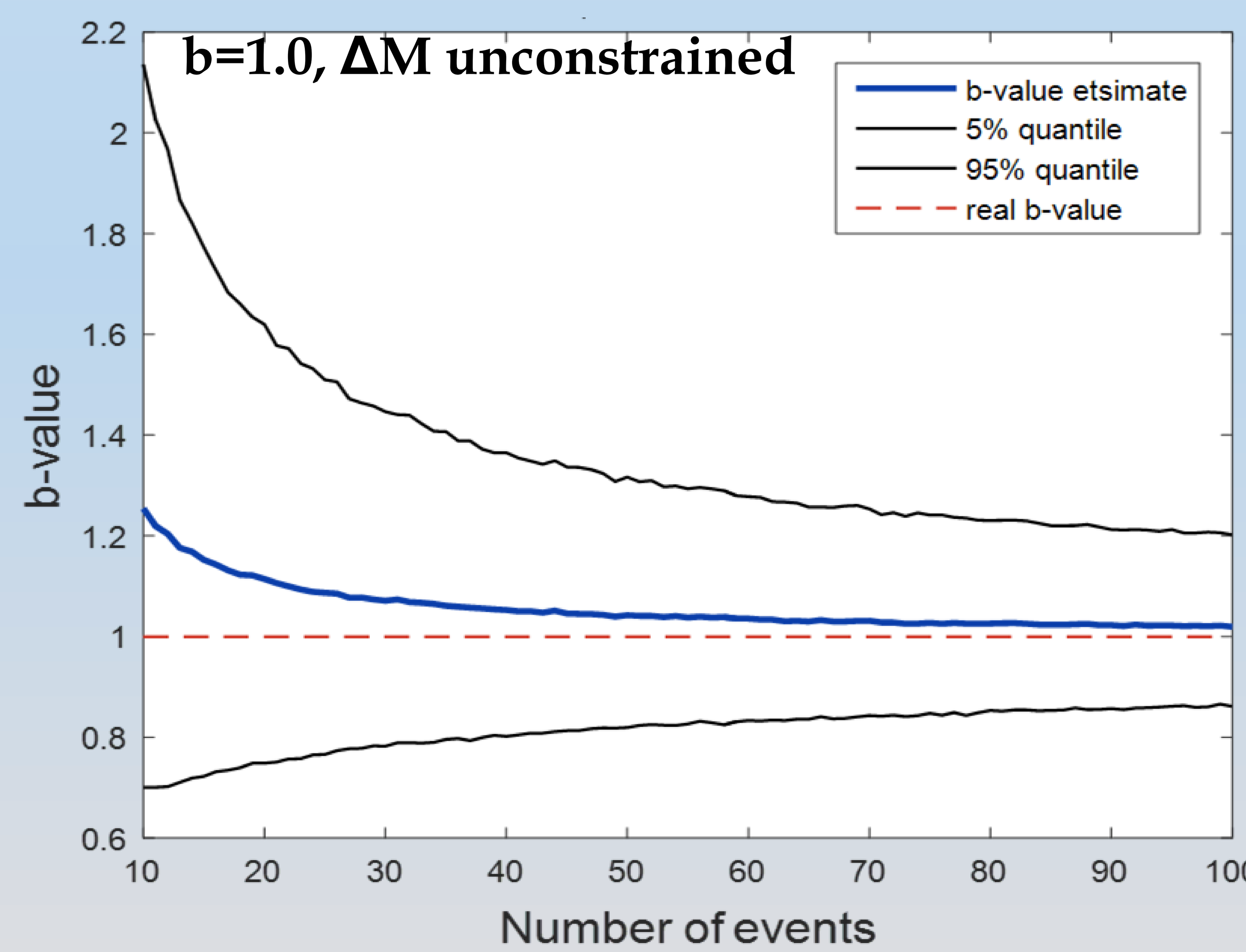
Detection function for $M<M_{min}$

❖ *b-value: estimated by Aki (1965) estimator*

❖ *M_C : found with the modified Goodness of Fit Test (mGFT, Leptokaropoulos et al., 2013; 2018)*

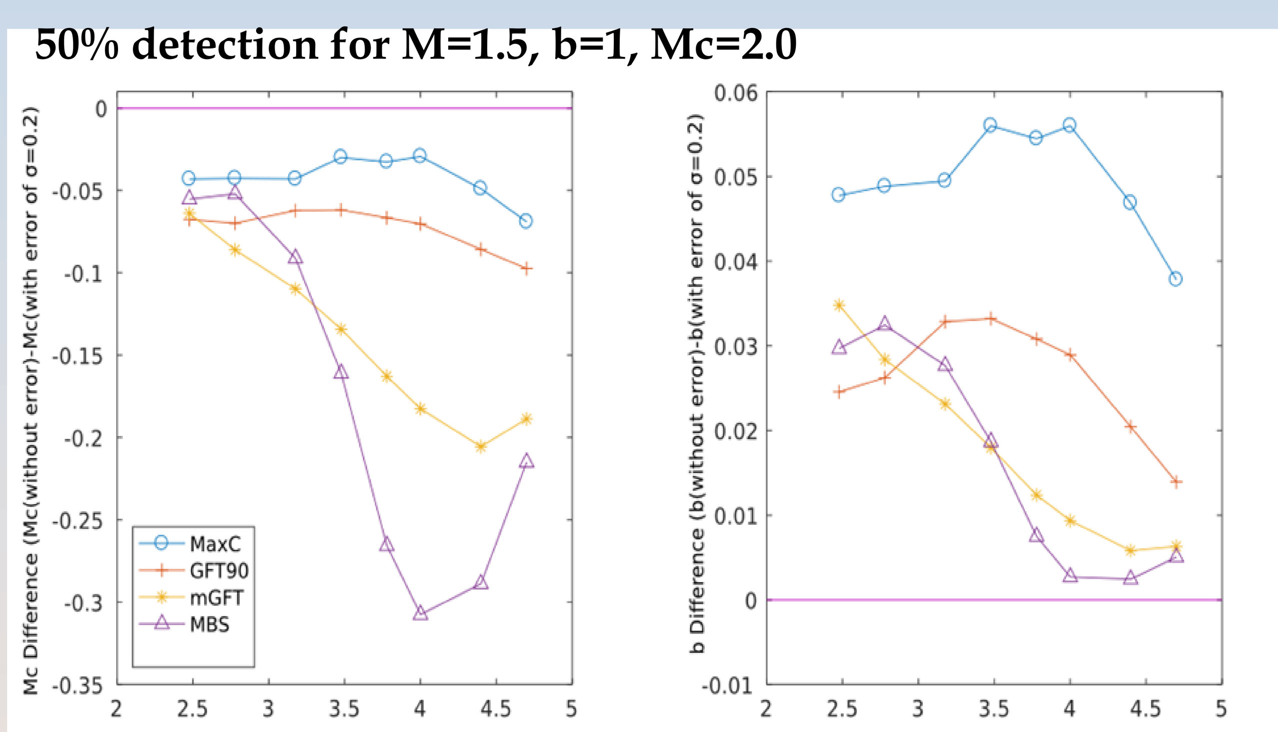


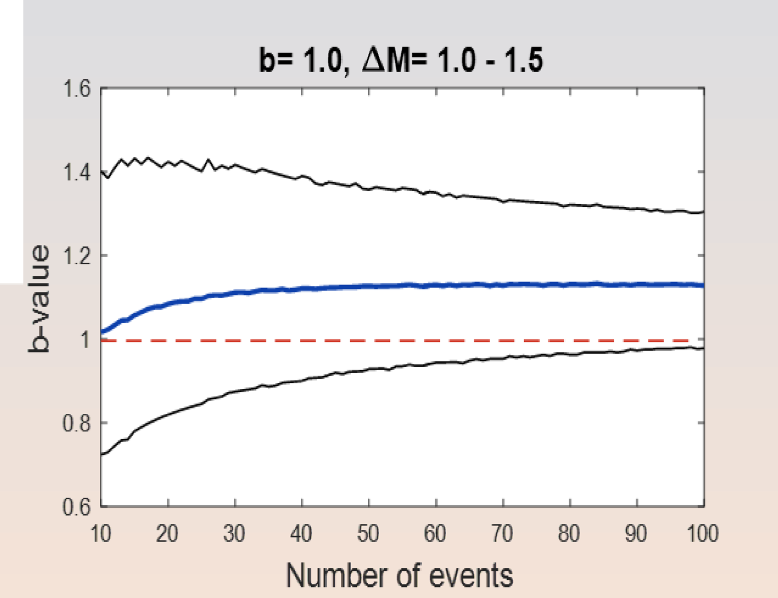


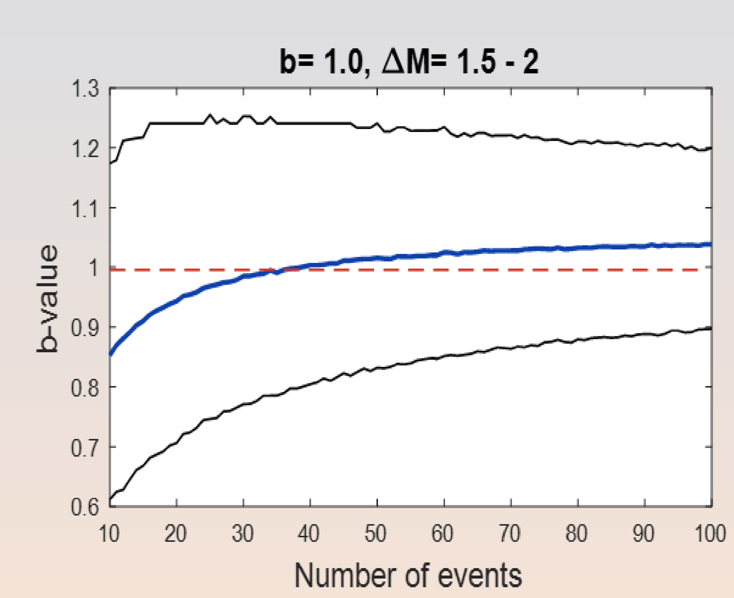


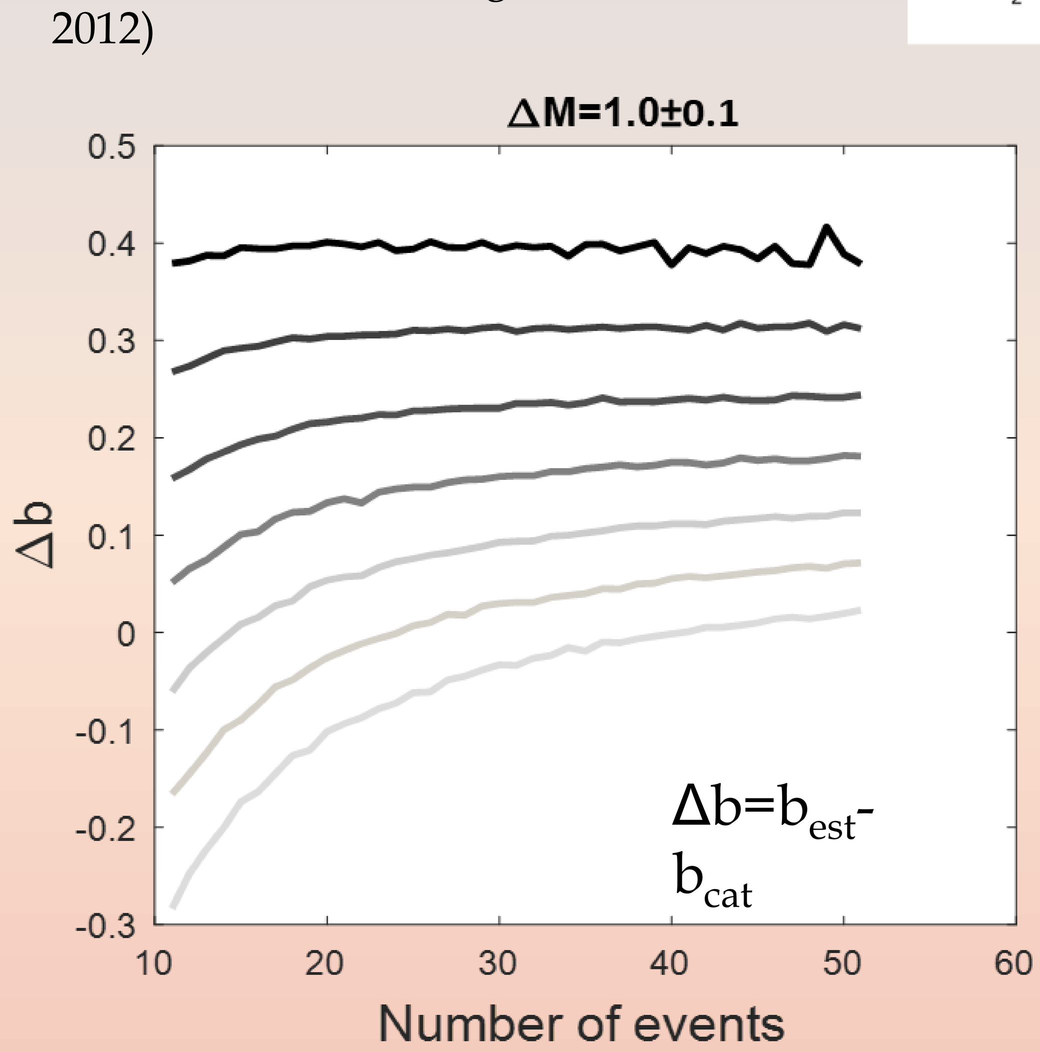
Nomograms can (and should) be constructed for any combination of:

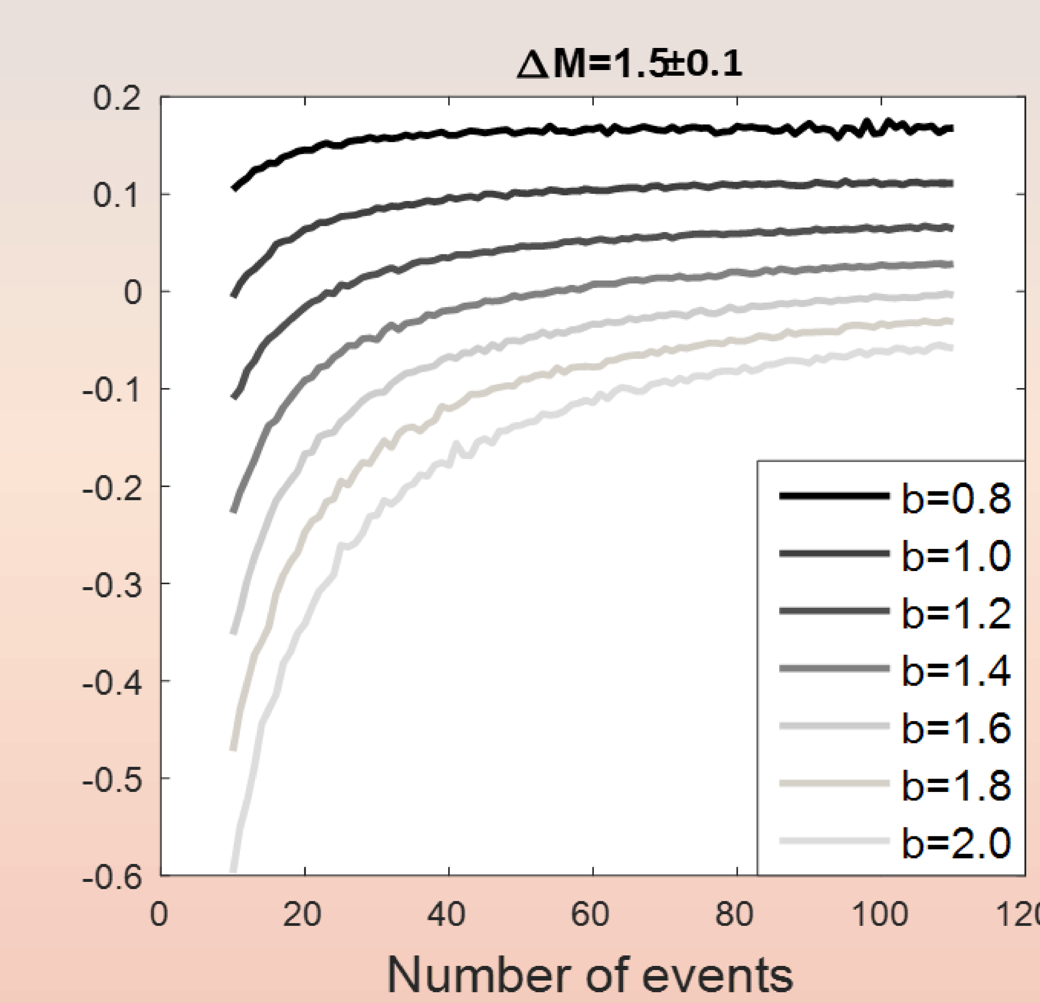
- ✓Magnitude distribution (b-value)
- ✓Magnitude range, ΔM
- ✓Catalogue size, N
- ✓Round-off interval (magnitude precision)
- ✓Magnitude uncertainties (noise of selected distribution and parameters)
- ✓Finite of infinite M_{max} (Unlimited or Truncated G-R)
- ✓M_C estimation technique (e.g. Woessner and Wiemer, 2005; Mignan and Woessner, 2012)

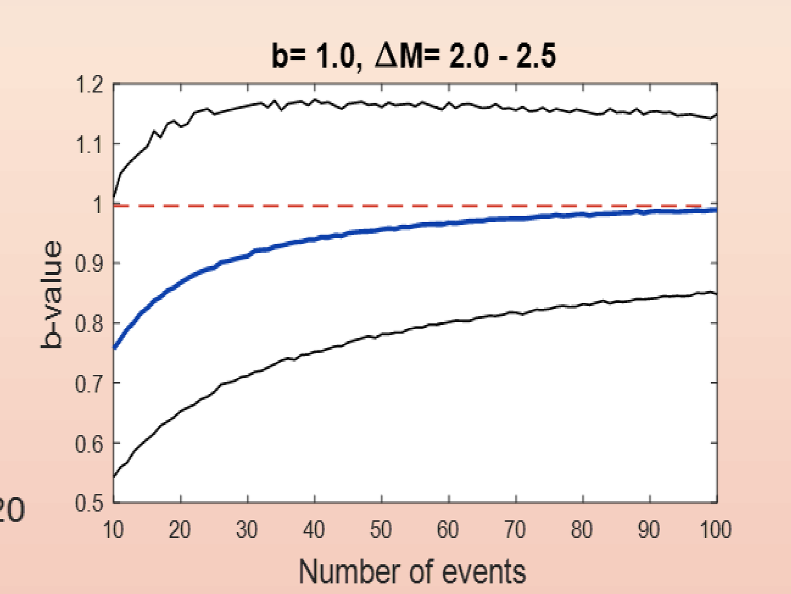


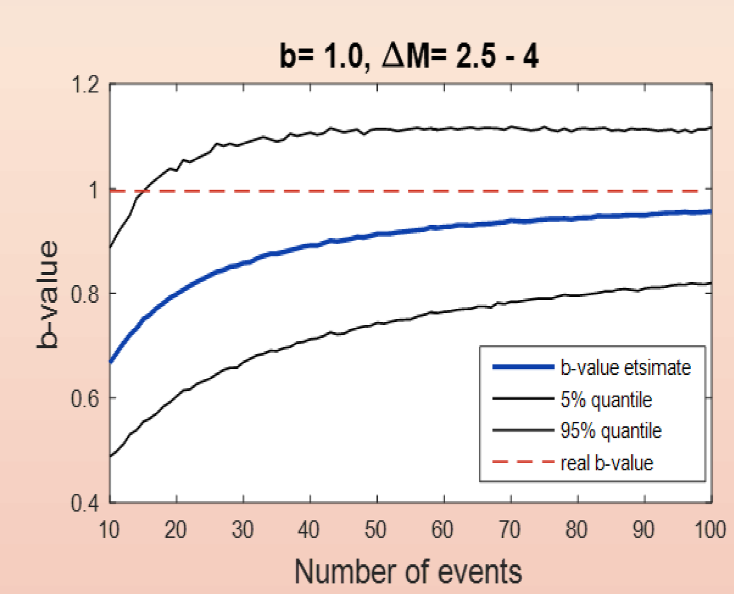












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SHale gas **E**xploration and **E**xploitation induced **R**isks

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